Multiple-Choice Questions
Please circle only one answer.

1. **[2 marks]** Let $f(x) = x^3 + \sqrt{x}$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.

   (a) $f'(1) = 0$
   (b) $f'(1) = 1.5$
   (c) $f'(1) = 3.5$
   (d) $f'(1) = 6$

2. **[2 marks]** Let $f(x) = \ln(\sqrt{x})$. Evaluate $f''(2)$. In other words, find the second derivative of f at $x = 2$.

   (a) $f''(2) = -\dfrac{1}{8}$
   (b) $f''(2) = 0$
   (c) $f''(2) = -1$
   (d) $f''(2) = \dfrac{3}{4}$

3. **[2 marks]** Let $f(x) = |x - 1|$. Calculate

$$L = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}.$$

   (a) $L = 0$
   (b) This limit does not exist
   (c) $L = -1$
   (d) $L = 1$

4. **[2 marks]** Evaluate the following limit: $L = \lim_{x \to \infty} x \sin\left(\dfrac{1}{x}\right)$.

   (a) $L = 1$
   (b) $L = 4$
   (c) $L = 2$
   (d) $L = -1$

5. **[2 marks]** A differentiable function f has the property that $f(1) = 6$, $f'(6) = -2$ and $f'(1) = 3$. What is the value of the derivative of $f(f(x))$ at $x = 1$?

   (a) -3
   (b) -6
   (c) x
   (d) 2

6. **[2 marks]** Let $f(x) = \tan(1 + \sin x)$. Evaluate $f'(x)$. In other words, find the derivative of f at x.

   (a) $f'(x) = -\sin x \ \sec^2(\cos x)$
   (b) $f'(x) = \sec^2(1 + \sin x)$
   (c) $f'(x) = \sec^2(\sin x)$
   (d) $f'(x) = \cos x \ \sec^2(1 + \sin x)$

7. **[2 marks]** Let y be given implicitly as a differentiable function of x by $x^2 \cos y + y^2 - 1 = 0$. Then the slope of the tangent line to the curve $y = y(x)$ at the point (x, y) where $x = 0$, $y = 1$ is equal to:

   (a) 2,
   (b) $+\infty$,
   (c) $\dfrac{1}{2}$,
   (d) 0.

8. **[2 marks]** Let $f(x) = (x + 1)^{3x}$. Evaluate $f'(0)$. In other words, find the derivative of f at $x = 0$.

   (a) $f'(0) = 0$
   (b) $f'(0) = 1$
   (c) $f'(0) = -1$
   (d) $f'(0) = 12$

9. **[2 marks]** Let $f(x) = \text{Arcsin}\left(\cos x^2\right)$. Calculate $f'(x)$ in the case where x is a point where $\sin(x^2) > 0$.

   (a) $f'(x) = -5$
   (b) $f'(x) = x$
   (c) $f'(x) = -2x$
   (d) None of these

10. **[2 marks]** Evaluate $L = \lim_{x \to 0} \left(\dfrac{\sin 4x}{\sin 2x} \right)$ using any method.

    (a) $L = 1$
    (b) $L = 1.85$
    (c) $L = 2$
    (d) $L = \dfrac{5}{2}$

11. **[2 marks]** Evaluate

$$\lim_{x\to+\infty} \frac{d}{dx} \int_{\sqrt{3}}^{\sqrt{x}} \frac{r^3}{(r+1)(r-1)}\, dr$$

(a) $I = 0$

(b) $I = \dfrac{27}{2}$

(c) This limit does not exist

(d) $I = \dfrac{1}{2}$

12. **[2 marks]** Which of the following functions F represents the form of the inverse function of the function $f(x) = \sqrt{x^2 - 4}$ whose domain is the set of all real numbers x where $x \geq 2$.

(a) $F(x) = \sqrt{x^2 + 4}$ where $Dom(F) = \{x \ : \ 0 \leq x < +\infty \}$

(b) $F(x) = -\sqrt{x^2 + 4}$ where $Dom(F) = \{x \ : \ -\infty < x < +\infty \}$

(c) $F(x) = \sqrt{4 + x^2}$ where $Dom(F) = \{x \ : \ -\infty < x < +\infty \}$

(d) $F(x) = \sqrt{x^2 + x + 1}$

13. **[2 marks]** Solve the inequality $x^2 - 3x + 2 < 0$ for x.

(a) $\{x : -2 < x < -1\}$

(b) $\{x : 1 < x < 2\}$

(c) $\{x : -\infty < x < 1\}$

(d) $\{x : 2 < x < \infty\}$

14. Determine an interval where the graph of the function defined by the polynomial $p(x) = x^4 - 6x^3 + 12x^2$ is concave up.

(a) $\{x : -5 < x < 0\}$

(b) $\{x : -\infty < x < 1.78\}$

(c) $\{x : 2 < x < \infty\}$

(d) $\{x : 1 < x < 2\}$

15. **[2 marks]** Which of the following functions has a point of inflection at $x = 0$?

    (a) $f(x) = x^2 + x + 1$

    (b) $f(x) = -x^2 - 4$

    (c) $f(x) = 2x^3 + 6$

    (d) $f(x) = x^4 + 12$

16. **[2 marks]** For what values of x is the function $f(x) = \dfrac{1}{x^2 - 1}$ increasing?

    (a) $x > 0$

    (b) $x < 0$

    (c) $0 < x < 1$

    (d) $x > 1$

17. **[2 marks]** Find all the critical points of $f(x) = x^3 - 3x + 2$

    (a) $x = -1,\ x = 1$

    (b) $x = 0,\ x = 1$

    (c) $x = -2, x = 0$

    (d) $x = 0,\ x = 1,\ x = 2$

18. **[2 marks]** An antiderivative of $f(x) = \cos(3x + 6)$ is given by

    (a) $\sin(3x + 6)$

    (b) $-3\sin(3x + 6)$

    (c) $\dfrac{\sin(3x + 6)}{3}$

    (d) $\sin(3x^2 + 6x)$

19. **[2 marks]** Evaluate $\int x^2\ 3^{2x^3+1}\ dx$

    (a) $\dfrac{\ln 3\ 3^{2x^3+1}}{6}$

    (b) $6x^2\ 3^{2x^3+1}$

    (c) 3^{2x^3+1}

    (d) $\dfrac{3^{2x^3+1}}{6\ln 3}$

20. **[2 marks]** Evaluate $\int_0^4 x\ \sqrt{2x+1}\ dx$

    (a) 9

    (b) $\dfrac{298}{15}$

    (c) $\dfrac{3}{4}$

    (d) $\dfrac{1}{2}$

21. **[2 marks]** The value of $\int \dfrac{dx}{x\ln x}$ is

    (a) $\ln(\ln x)\ +\ C$

    (b) $\ln\ln|x|\ +\ C$

    (c) $\ln|x| + C$

    (d) $\ln|\ln x\ | + C$

22. **[2 marks]** The most general antiderivative of $x^2\ e^{3x}$ is given by

    (a) $\dfrac{1}{3}x^2\ e^{3x} - \dfrac{2}{9}x\ e^{3x} + \dfrac{2}{27}\ e^{3x}\ +\ C$

    (b) $\dfrac{1}{9}x^3\ e^{3x}\ +\ C$

    (c) $\dfrac{1}{9}x^2\ e^{3x} + C$

    (d) $\dfrac{1}{3}x^2\ e^{3x} - \dfrac{2}{9}x\ e^{3x} + C$

23. **[2 marks]** Evaluate and simplify the indefinite integral: $\int e^{2x} \sin 3x \, dx$.

(a) $-\frac{9}{13} e^{2x} \cos 3x - \frac{2}{13} e^{2x} \sin 3x + C$

(b) $\frac{1}{13} e^{2x} \cos 3x + \frac{1}{13} e^{2x} \sin 3x + C$

(c) $\frac{2}{13} e^{3x} \cos 2x + \frac{9}{13} e^{3x} \sin 2x + C$

(d) $-\frac{3}{13} e^{2x} \cos 3x + \frac{2}{13} e^{2x} \sin 3x + C$

24. **[2 marks]** Evaluate the improper integral $\int_0^\infty x^3 e^{-x} \, dx$.

(a) 6

(b) -8

(c) 0

(d) 1

25. **[2 marks]** The form of the partial fraction decomposition of the rational function

$$\frac{x-1}{(x^2+1)(x^2-2x-3)}$$

is

(a) $\frac{A}{x^2+1} + \frac{C}{x-3}$

(b) $\frac{Ax+B}{x^2+1} + \frac{C}{x-3} + \frac{D}{x+1}$

(c) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-3}$

(d) $\frac{Ax+B}{x^2+1} + \frac{C}{x+3} + \frac{D}{x+1}$

where A, B, C, D are constants to be determined.

26. **[2 marks]** Evaluate the indefinite trigonometric integral

$$\int \sin^3 x \ \cos^2 x \ dx.$$

(a) $\dfrac{\sin^4 x \ \cos^2 x}{4} + \ C$

(b) $\dfrac{\sin^3 x \ \cos^3 x}{3} + \ C$

(c) $\dfrac{\cos^5 x}{5} - \dfrac{\cos^3 x}{3} + \ C$

(d) $\dfrac{\sin^5 x}{5} - \dfrac{\sin^3 x}{3} + \ C$

where A, B, C, D are constants to be determined.

27. **[2 marks]** The area enclosed by the intersection of the two curves defined by $y = 1 - x$ and $y = 2x^2$ is given by which of the following definite integrals?

(a) $\displaystyle\int_{-1}^{\frac{1}{2}} (2x^2 - 1) \ dx$

(b) $\displaystyle\int_{-1}^{\frac{1}{2}} \left(1 - x - 2x^2\right) \ dx$

(c) $\displaystyle\int_{-1}^{\frac{1}{2}} \left(2x^2 - x + 1\right) \ dx$

(d) $\displaystyle\int_{-1}^{2} \left(x - 2x^2\right) \ dx$

28. **[2 marks]** Which of the following expressions gives the **volume** of the solid of revolution obtained when the region bounded by the graphs of $y = 2x$ and $y = 4x^2$ is revolved about the $\boldsymbol{y}$**−axis**?

(a) $I = \displaystyle\int_0^{\frac{1}{2}} 2\pi(2x - 4x^2) \ dx$

(b) $I = \displaystyle\int_0^{\frac{1}{2}} 2\pi x \ (4x^2 - 16x^4) \ dx$

(c) $I = \displaystyle\int_0^{\frac{1}{2}} 2\pi x \ (2x - 4x^2) \ dx$

(d) $I = \displaystyle\int_0^{1} 2\pi y \ (\sqrt{y} - y) \ dy$

29. **[2 marks]** The center of mass of a thin quarter circle of radius R having uniform density and situated in the first quadrant is given by which of the following points?

(a) $\left(\frac{4\pi}{R}, \frac{4\pi}{R}\right)$

(b) $\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$

(c) $\left(\frac{\pi}{R}, \frac{\pi}{R}\right)$

(d) $\left(\frac{R}{\pi}, \frac{R}{\pi}\right)$

30. **[2 marks]** The general solution of the differential equation

$$y^2 \frac{dy}{dt} = t^2 e^{-y^3} \ln t$$

is given by,

(a) $\frac{1}{3}e^{y^3} = \frac{1}{3}t^3 \ln t - \frac{t^3}{9} + C$

(b) $e^{y^3} = \frac{1}{2}t^2 \ln 2t + \frac{1}{9}e^t + C$

(c) $\frac{1}{3}e^{y^3} = \frac{1}{2}t^2 \ln t - t + C$

(d) $e^{y^2} = \frac{1}{3}t^3 \ln t - \frac{t^3}{9} + C$

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